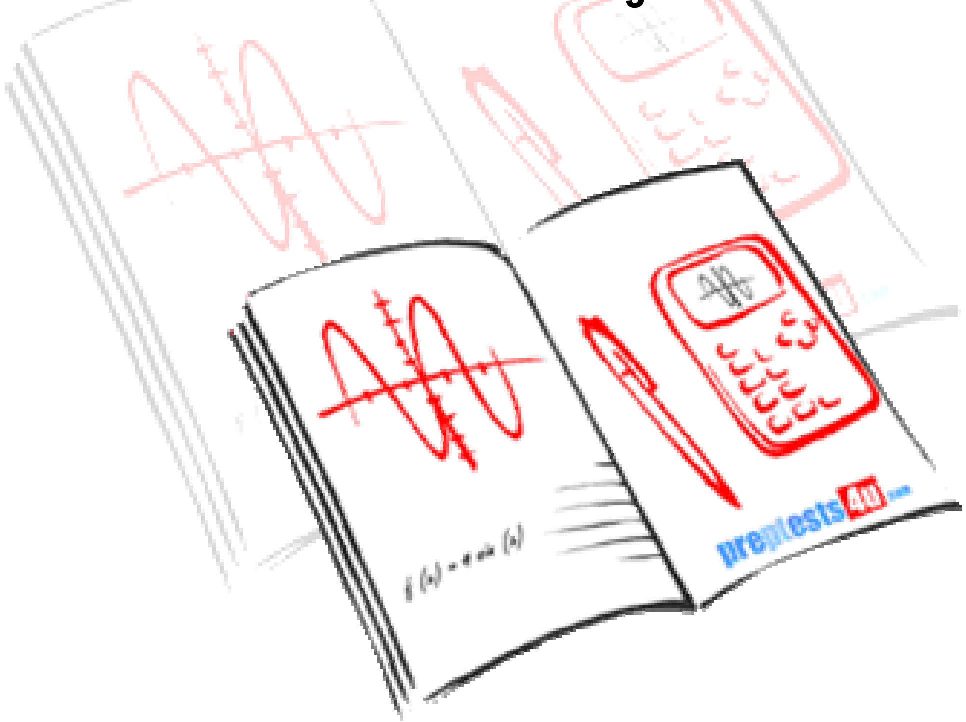
Calculus 1

Chapter 4 - The Concept of Integration

Definitions of Indefinite and Definite Integrals



Indefinite and Definite Integrals

Indefinite Integrals

Function $F(x)$ is anti-derivative or indefinite integral of $f(x)$ on a given interval if $F^{'}(x) = f(x)$. This is written as

$$\int f(x)dx = F(x) + C$$

Since C is a constant (independent of x), indefinite integral of function $f(x)$ is not unique.

Definite Integrals

In this case the upper and lower limits of integration are given as b and a ,respectively. Assuming that the function $f(x)$ is continuous on $[a,b]$, hence,

$$\int_a^b f(x)dx = F(b) - F(a)$$

The result of the above integral would be a real number. However, the limits of integration are not necessarily constant values as explained in the following examples.

Example 1:

$$\int (x^3 - 3x^2 + 1)dx = \frac{1}{4}x^4 - x^3 + x + C$$

Example 2:

$$\int_0^1 (x^3 - 3x^2 + 1)dx = \left[\frac{1}{4}x^4 - x^3 + x\right]_0^1 = \frac{1}{4}$$

Note:
Function $f(x) = x^3 - 3x^2 + 1$ is continuous on $[0,1]$.

Example 3:

$\int_{-1}^{1} \frac{1}{x} dx$ does not satisfy the condition of continuity since $f(x) = \frac{1}{x}$ is not continuous at $x = 0$. It is relatively easy to find an approximate answer.

Example 4:

$\int_{0}^{2} \sqrt{(x^2 - 1)} dx$,

Function $f(x) = \sqrt{(x^2 - 1)}$ is not defined on $[0,1)$, hence not continuous on $[0,2]$.

Some Basic Formulas of Integration

1. $\int (u + v) dx = \int u dx + \int v dx$

2. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

3. $\int \frac{du}{u} = Ln|u| + C$

4. $\int e^u du = e^u + C$

5. $\int a^u du = \frac{1}{Lna} a^u + C$

6. $\int_{g(t)}^{h(t)} f(x) dx = f(h(x)) * \frac{dh}{dx} - f(g(x)) * \frac{dg}{dx}$, if h and g are constant, then the result is 0

7. $\int u dv = uv - \int v du$