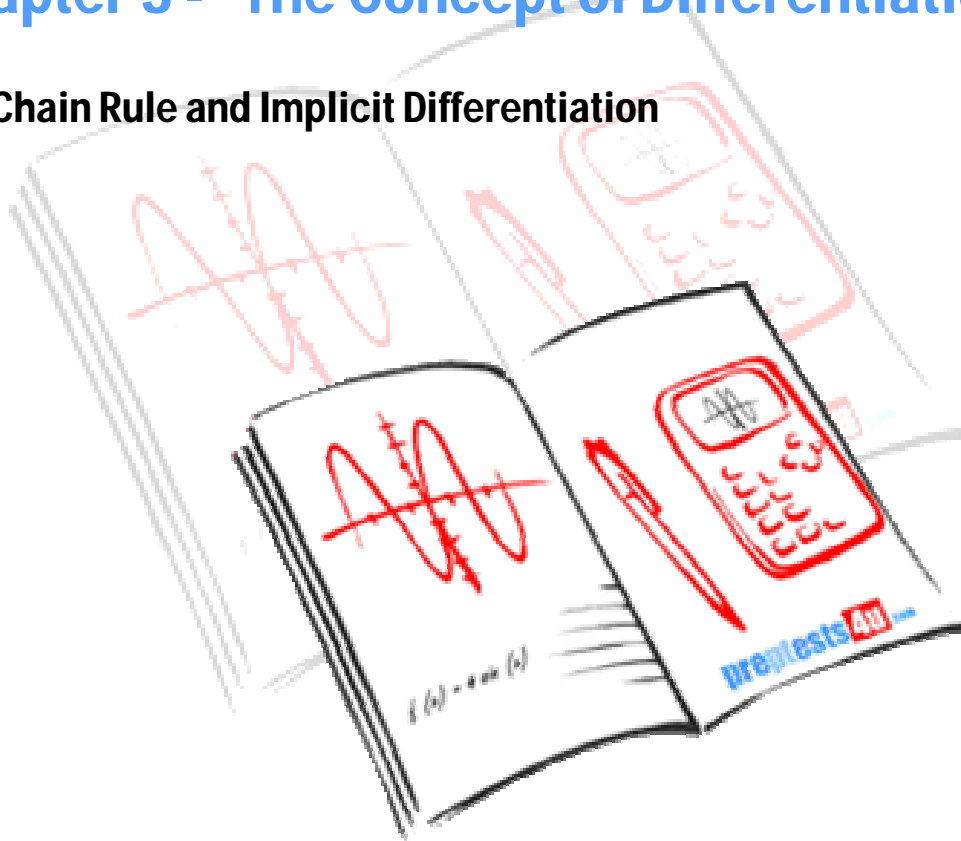


# Calculus 1

## Chapter 3 - The Concept of Differentiation

### The Chain Rule and Implicit Differentiation



## The Chain Rule and Implicit Differentiation

Assuming  $y = f(g)$  and  $g = g(x)$ , hence  $y = f(g(x))$  and the derivative of  $y$  with respect to  $x$ , called chain rule, is as follow:

$$\frac{dy}{dx} = \frac{dy}{dg} * \frac{dg}{dx}$$

### Example 1:

Given

$$h(x) = (f \circ g)(x) = f(g(x)) \text{ and } g(4) = 2, g'(4) = 1, f'(2) = 5, f(4) = 6$$

Find  $h(4) = ?$

Solution:

$$h'(x) = \frac{dh}{dx} = \frac{df}{dg} * \frac{dg}{dx}$$

$$h'(4) = f'(g(4)) * g'(4) = f'(2) * g'(4) = 5 * 1 = 5$$

### Example 2:

Given  $f(x) = 2\cos u + 1$  and  $u = \sqrt{1+x}$ , find  $f'(x)$ .

Using Chain Rule formula:

$$\frac{df}{dx} = \frac{df}{du} * \frac{du}{dx} = (-2\sin u) * \frac{1}{2\sqrt{1+x}} = \frac{-2\sin u}{2\sqrt{1+x}} = \frac{-\sin\sqrt{1+x}}{\sqrt{1+x}}$$

Exercise: Find  $f'(-1)$

## Implicit Differentiation

### Example 3:

Find the equation of the tangent line to the curve  $xy^2 + 2 = \frac{x^2}{y}$  at  $(2,1)$ .

#### Solution:

Before finding the derivative of the given equation, make sure that the given point is on the curve; hence, substitute  $x$  and  $y$  values in the equation. Next take the derivative of both sides of the equation with respect to  $x$  as follow: Multiplying both sides by  $y$  and taking derivative,

$$xy^3 + 2y = x^2$$

$$(1)(y^3) + (x)(3y^2 y') + 2(y') = 2x$$

Rearranging the above equation and solving for  $y'$ ,

$$y' = \frac{2x - y^3}{2 + 3xy^2}$$

$$\text{Slope} = m = y'(2,1) = \frac{2 * 2 - 1^3}{2 + 3 * 2 * 1^2} = \frac{3}{8}$$

Equation of the tangent line is:

$$y - 1 = \frac{3}{8}(x - 2) \Rightarrow y = \frac{3}{8}x + \frac{1}{4}$$

### Example 4:

Find the derivative of the following equation and determine the slope and equation of tangent line at  $(0,0)$ .

$$y^4 - x^3 y^2 = x - x^2 y^3$$

#### Solution:

Taking derivative of both sides with respect to  $x$ ,

$$4y'y^3 - 3x^2y^2 - 2y'yx^3 = 1 - 2xy^3 - 3y'y^2x^2$$

Rearranging the terms and solving for  $y'$ ,

$$y' = \frac{dy}{dx} = \frac{1 - 2xy^3 + 3x^2y^2}{4y^3 - 2yx^3 + 3y^2x^2}$$

$$\text{Slope} = m = y'(0,0) = \frac{1}{0} = \infty = \text{undefined}$$

**Equation of the tangent line at  $(0,0)$  with slope  $\infty$  is the line  $x = 0$ , or Y-Axis.**

**Example 5:**

Find the derivative of the given equation and show if the function is differentiable at point  $(0,0)$

$$\sin(xy) = e^{(xy)}$$

**Solution:**

$$y\cos xy + xy'\cos xy = ye^{-xy} + xy'e^{-xy}$$

$$y' = \frac{dy}{dx} = \frac{ye^{-xy} - y\cos xy}{x\cos xy - xe^{-xy}} = \frac{-y}{x}$$

**Exercise:** Check for the differentiability of the given function at  $(0,0)$ .