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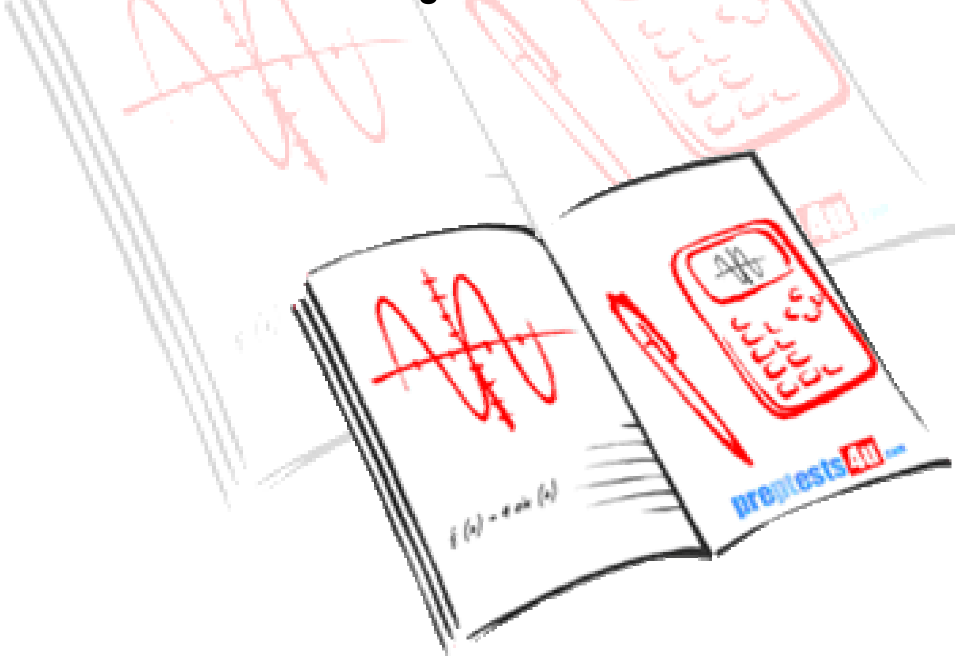
Calculus 1

Chapter 2 - The Concepts of Limit and Continuity

The Limits of Functions of Single Variable



The Limits of Functions of Single Variable

Example 1:

$$\lim_{x \to 2}(3x+5)=11 \quad , \quad \lim_{x \to 2^-}(3x+5)=11 \quad , \quad \lim_{x \to 2^+}(3x+5)=11$$

Example 2:

$$\lim_{x \to 1}(2x^2-x+1)=2 \quad , \quad \lim_{x \to 1^-}(2x^2-x+1)=2 \quad , \quad \lim_{x \to 1^+}(2x^2-x+1)=2$$

Note:

As you may have noticed, limits of left and right are the same for polynomials and straight lines.

Example 3:

$$\lim_{x \to 0}\left(\frac{x-1}{x^2+1}\right)=-1$$

Example 4:

$$\lim_{x \to 2}\left(\frac{-6x+3x^2}{x^2-4}\right)=\frac{0}{0}=\frac{0}{0} \quad , \quad \lim_{x \to 2}\frac{(3x)(x-2)}{(x-2)(x+2)}=\frac{6}{4}=\frac{3}{2}$$

Note:

Anytime limit approaches $\frac{0}{0}$, this means that the numerator and denominator might have common factors. In some cases, as this one, one can remove the factor which is causing the numerator and denominator to vanish. In other cases, such rectification may not be possible by factoring, as indicated by the next example.

The common factor $(x-2)$ has been cancelled from top and bottom since $x \to 2$ and not $x = 2$.

Example 5:

$$\lim_{x \to 1} \left(\frac{x^e - 1}{x - 1} \right) = \frac{0}{0} \text{ .}$$

As you see, it is not possible to do factoring and cancel the common factor(s) causing $\frac{0}{0}$.

This problem is resolved by using **l'Hospital's Rule**. In order to use this rule, one must know the concept of derivatives, **covered in chapter 3**. So, for the time being, simply accept the fact that the answer to the above limit is **irrational number** e.

Example 6:

$$\lim_{x \to 0} \frac{\sqrt{x-1}}{x+1} = Undefined$$

Since the domain for this function is $(1, \infty)$

Example 7:

$$\lim_{x \to \infty} \frac{\sqrt{x-1}}{x+1} = \lim_{x \to \infty} \frac{\sqrt{x}}{x} = \lim_{x \to \infty} \frac{1}{\sqrt{x}} = 0$$

Example 8:

$$\lim_{x \to 1^-} \frac{\sqrt{x-1}}{x+1} = Undefined$$

Since $x \to 1^-$ from the left is making the radical undefined.

Example 9:

$$\lim_{x \to 1^+} \frac{\sqrt{x-1}}{x+1} = 0$$

Since $x \to 1^+$ from right, the value of x is always larger than 1 making the radical defined

Hence it is possible to have only one side limit, left or right, to exist. Indeed, limit of the function given in the Example 9 as $x \to 1$ **does not exist**.

Example 10:

$$\lim_{x \to 0}\left(\frac{-2x^2+3x-1}{|x|}\right) = ?$$

This problem is somehow different from the previous ones since the absolute value given in the denominator takes two possible choices:

$$|x| = x \qquad x \geq 0$$

$$|x| = -x \qquad x < 0$$

Therefore, we need to examine the left and right limits before answering the question posed.

$$\lim_{x \to 0^+}\left(\frac{-2x^2+3x-1}{x}\right) = -\infty$$

$$\lim_{x \to 0^-}\left(\frac{-2x^2+3x-1}{-x}\right) = +\infty$$

Once one of the one side limit, left or right, does not exist, it is not necessary to check for both left and right limits. In this case, limit of the given function in Example 10 **does not exist**.

Example 11:

$$\lim_{x \to \infty}\left(e^{-x+1}-1\right) = \lim_{x \to \infty}\left(e^{-x+1}\right) - \lim_{x \to \infty} 1 = e\lim_{x \to \infty} e^{-x} - 1 = -1$$

Example 12:

$$\lim_{x \to 3} \log\left(9 - x^2\right) = Undefined$$

The domain for this function is $(-3,+3)$. If $x \to 3^+$, from right side, then logarithm is undefined.

If $x \to 3^-$, from left side, then the value inside parentheses approaches zero and logarithm of zero is undefined. This is an example of having left and right side limits to be undefined.
So far, you have had enough of limits or should we say that you are up to your limits! More interesting materials are brought to you in the following sections.