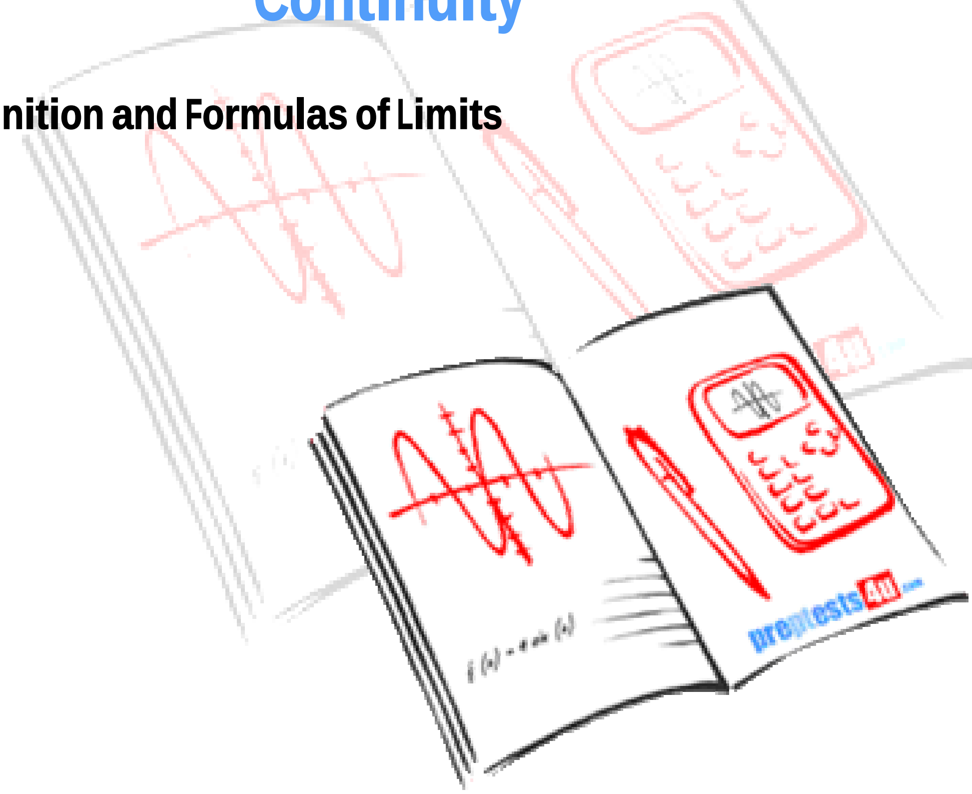
Calculus 1

Chapter 2 - The Concepts of Limit and Continuity

Definition and Formulas of Limits



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Definition and Formulas of Limit

Definition of Limit

a. $\lim_{x \to a} f(x) = A$ if $f(x)$ approaches the value of A when x is sufficiently close to the value of a, but not equal to a.

b. $\lim_{x \to a^-} f(x) = A_1$ is called limit from left since x is approaching a from left side.

c. $\lim_{x \to a^+} f(x) = A_2$ is called limit from right since x is approaching a from right side.

d. $\lim_{x \to a} f(x) = A$ **exist if and only if** $\lim_{x \to a^-} f(x) = A_1 = \lim_{x \to a^+} f(x) = A_2 = A$

Some Useful Formulas

Let $\lim_{x \to a} f(x) = A$ and $\lim_{x \to b} g(x) = B$.

a. $\lim f(x) = C$, a constant, $\lim_{x \to a} f(x) = C$

b. $\lim_{x \to a} [\, f(x) \pm g(x) \,] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = A \pm B$

c. $\lim_{x \to a} [\, f(x) * g(x) \,] = \lim_{x \to a} f(x) * \lim_{x \to a} g(x) = A * B$

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d. $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = \frac{A}{B}$$, provided $B \neq 0$

e. $$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{A} = A^{1/n}$$

Note:
If $A < 0$ and n is a positive integer, then such limit does not exist.

Limit Rules for Rational Functions as $x \to \infty$

In Chapter 1. we defined Rational Functions to be a fractional equation which numerator and denominator are both polynomials of some degrees, $f(x) = \frac{R(x)}{S(x)}$

In order to evaluate $\lim\limits_{x \to \infty} f(x)$ or $\lim\limits_{x \to -\infty} f(x)$, we need to look into three cases:

a. **Highest degree of $R(x) <$ highest degree of $S(x)$**

$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0$$

Example 1:

$$f(x) = \frac{3x^2 - 4x + 1}{x^3 - 2x + 5}$$

$$\lim_{x \to \pm\infty} f(x) = 0$$

since the highest degree of the numerator is 2 and the highest degree of the denominator is 3. **The Horizontal Asymptote for this function is line $y = 0$, which is X-Axis.**

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b. Highest degree of $R(x) >$ **highest degree of** $S(x)$

Example 2:

$$f(x) = \frac{3x^3 - 2x + 1}{x^2 - 4x - 5}$$

$\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to -\infty} f(x) = -\infty$. In this case, the

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{3x^3}{x^2} = \lim_{x \to \pm\infty} 3x = \pm\infty$$

Like previous case, there is a Horizontal Asymptote which is the straight line $y = 3x$. However, it is well possible to have a curve as **Horizontal Asymptote** rather than **straight line**.

c. Highest degree of $R(x) =$ **highest degree of** $S(x)$

Example 3:

$$f(x) = \frac{-6x - 3x^2}{x^2 - 4}$$

$$\lim_{x \to \pm\infty} f(x) = \frac{-3}{1} = -3$$

The value of the limit is equal to coefficient of the highest power, leading coefficient, in numerator divided to the coefficient of the highest power in the denominator. **Line** $y = -3$ **is Horizontal Asymptote.**

More examples, including applications of Limit, are given in the following sections. Students are advised to go though the examples very carefully and patiently and try to redo them on their own.

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