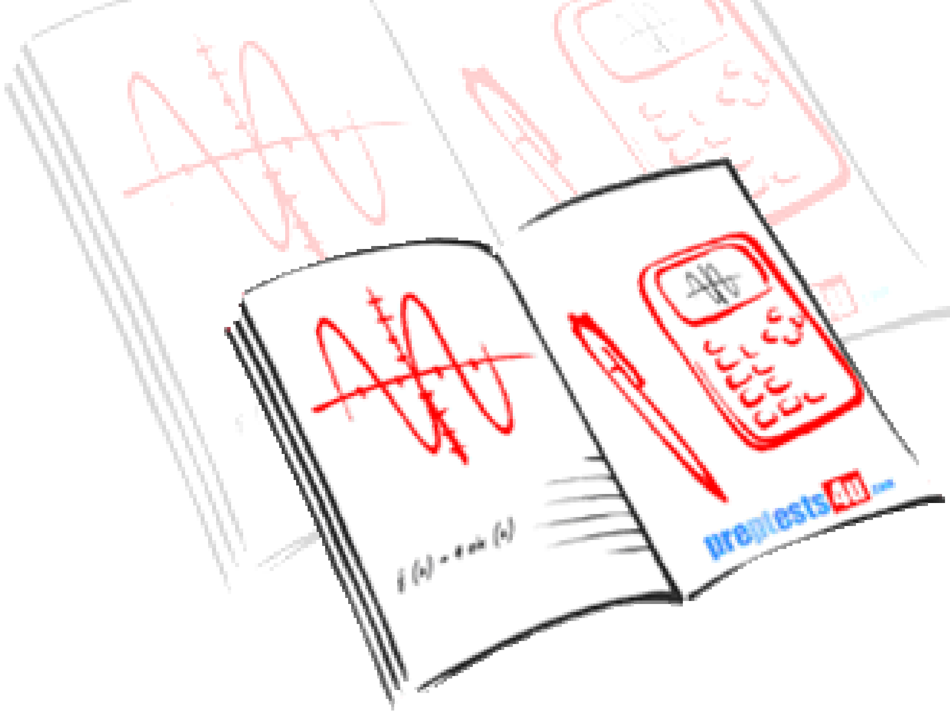
preptests4u.com

Calculus 1

Chapter 1 - Functions and Variables

Functions of One or More Variables



Functions of One or More Variables

Function of one variable has only one independent variable. For example, y = f (x) is a function of one variable, x, which might take any value in its domain and produce only one value for dependent variable, y.

On the other hand, the dependent variable might depend on two or more independent variables such as A = f(x,y) or V = f(x,y,z). However, Calculus 1 mainly focus on the functions of one and two variables.

The most common functions explored in this chapter are:

Polynomials

Given: $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x^1 + a_0$

where a's are the real numbers, called coefficients, and n's are nonnegative integers (i,e. zero or positive integers), called powers. The **highest power** is called the **degree** of the polynomial and the corresponding coefficient is called leading coefficient.

Example 1:

$f(x) = -5x^4 + 3x^2 - 2x + 1$ is a polynomial of **degree 4** with **leading coefficient (- 5).** ***The term with power three is missing***. This means that the coefficient for that term is zero. So, it is perfectly alright to have one or more terms missing in a polynomial, but having all of them missing will be a tragedy!!!

One of the frequently used polynomial is Quadratic Equation.
$f(x) = ax^2 + bx + c$. Graph of this function is concave up if **a** < **0**, and it is concave down if **a** > **0**. Coefficients **a**, **b**, and **c** are real numbers with the condition that $a \neq 0$.

Rational Functions

A function $f(x) = \dfrac{R(x)}{S(x)}$ is called rational function if both **R(x)** and **S(x)** are polynomials; otherwise it is called an algebraic equation. $f(x)$ is not defined for all the real value(s) where $S(x) = 0$ and $R(x) \neq 0$. **In case both R(x) and S(x) are zero for one or more values of x, then one should use the concept of limits to find the limiting value(s) of f(x). The value(s) at which** $S(x) = 0$ **and** $R(x) \neq 0$ **are called non-removable singularities (discontinuities).**

The vertical asymptotes of this function are located at the x-values where $S(x) = 0$ **and** $R(x) \neq 0$**.**

Example 2:

$$f(x) = \frac{4x^3 + 2x^2 - 1}{4 - x^2}$$

Find:

a. All the values in which $f(x)$ is not defined

$$4 - x^2 = 0, x = 2, x = -2$$

(*make sure you check these answers in the numerator*)

b. All the vertical asymptotes

Lines $x = 2$ and $x = -2$ are the vertical asymptotes.

c. Find all non-removable singularities (**discontinuity**)

$x = 2$ and $x = -2$ are **non-removable singularities**.

These values **do not** vanish the numerator; otherwise, one should apply the rules of limits to check for **removable singularities.**

Exponential Functions

$f(x) = a^x$ is called exponential function. **a** is the base and **x** is the exponent. Depending on the value of constant **a>0**, $f(x)$ could be either increasing or decreasing for all real values of independent variable x.

If a > 1, then $f(x)$ is an increasing function, or growth function; otherwise, it is a decreasing function, commonly called decay function.

Example 3:

Graph the function $f(x) = 3^x$ and label as increasing or decreasing function.

Solution:
By following the exponential function rule, this is an **increasing function, growth function,** since **a > 1**. In order to graph this function, we need to find the x-intercept, y-intercept, vertical and horizontal asymptotes (if any), domain, and range.

y-intercept,

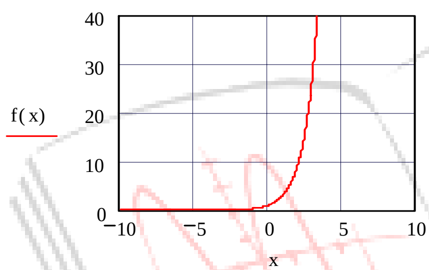
let x = 0, find y.

y = 1, this is y-intercept.

x-intercept,

let y = 0, find x.

As you observe there is no value of x which makes the value of y = 0; therefore, there is **no x-intercept**, meaning the graph does not cross x-axis. In the section of graph of functions, we will elaborate on other items needed for a reasonable graphing.



Example 4:

Graph the function $f(x) = \left(\frac{1}{3}\right)^x$ and label as increasing or decreasing function.

Solution:
By following the exponential function rule, this is an **decreasing function, decay function,** since **a < 1**. In order to graph this function, we need to find the x-intercept, y-intercept, vertical and horizontal asymptotes (if any), domain, and range.

y-intercept,

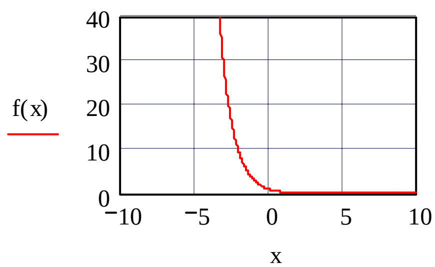
let x = 0, find y.

y = 1, this is y-intercept.

x-intercept,

let y = 0, find x.

As you observe there is no value of x which makes the value of y = 0; therefore, there is **no x-intercept**, meaning the graph does not cross x-axis. In the section of graph of functions, we will elaborate on other items needed for a reasonable graphing.



Logarithmic Functions

The function $f(x) = Log_a x$ is a logarithmic function where x > 0 and constant **a** > **0**. In fact, the graphs of logarithmic and exponential functions are symmetric with respect to the line y = x in the xy-plane. This function crosses x-axis at x = 1 but does not cross y-axis. For a constant value of **a > 1**, $f(x)$ is an increasing function. For **0 < a < 1**, the function $f(x)$ is decreasing.

Example 5 :

Graph $f(x) = Logx$, find x and y intercepts. Indicate if this function is increasing or decreasing.

Solution:

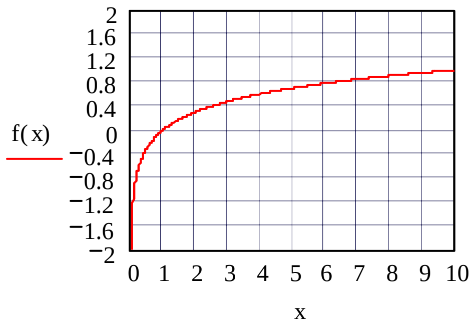
x-intercept,

let y = 0, find x.

x = 1, this is y-intercept.

y-intercept,

Since x > 0 (by definition) there is no y-intercept, meaning that $f(x)$ **does not cross y-axis**. This function is **increasing** according to the condition outlined above.



Example 6:

Graph $f(x) = -Log x$, find x and y intercepts. Indicate if this function is increasing or decreasing.

Solution:

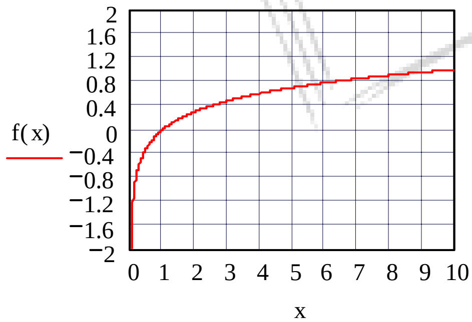
x-intercept,

let y = 0, find x.

x = 1, this is y-intercept.

y-intercept,

Since x > 0 (by definition) there is no y-intercept, meaning that $f(x)$ **does not cross y-axis**. This function is **decreasing** according to the condition outlined above.



Trigonometric Functions

A complete account of trigonometry functions is provided in Trigonometry Review materials available on the website. However, a quick review and some important properties of trigonometry functions are given below.

Fundamental Trigonometric Functions:

1	$f(x) = Sin(x)$	$-1 \leq Sin(x) \leq 1$	Period = 2π
2	$f(x) = Cos(x)$	$-1 \leq Cos(x) \leq 1$	Period = 2π
3	$f(x) = Tan(x)$		Period = π
4	$f(x) = Cot(x)$		Period = π
5	$f(x) = Sec(x)$		Period = 2π
6	$f(x) = Csc(x)$		Period = 2π

Graphs of these equations are given in the following sections.