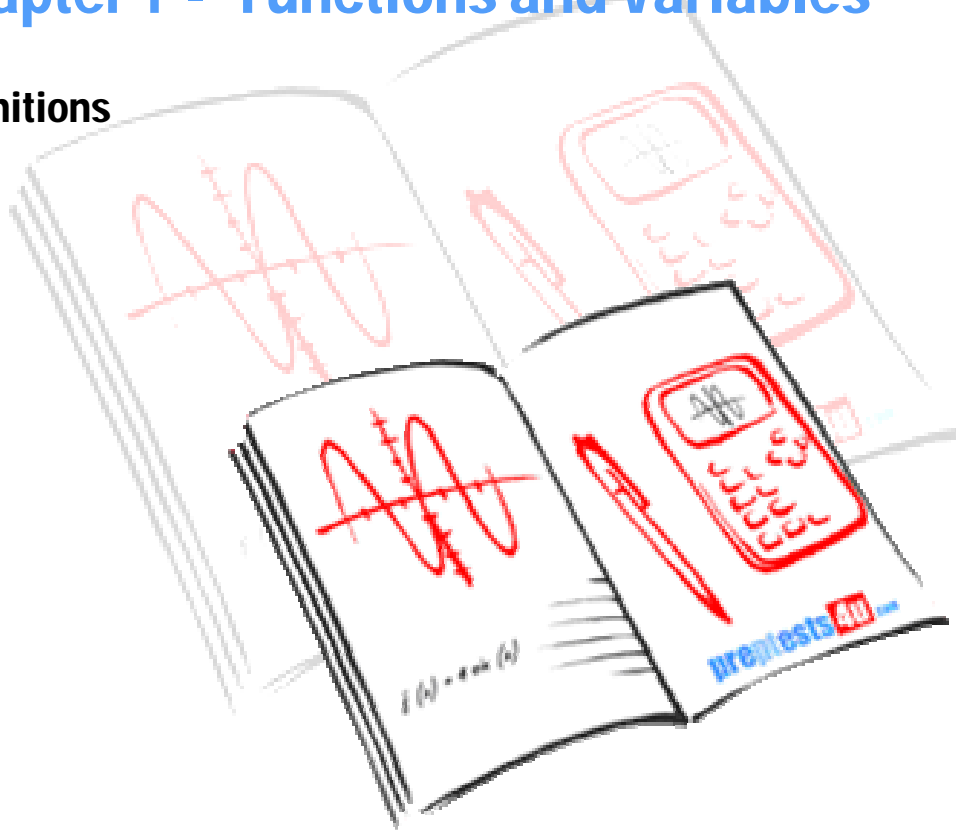


Calculus 1

Chapter 1 - Functions and Variables

Definitions



Definitions

A variable **y** is said to be a function of another variable **x** if there is a relation such that for each value of **x** (in its domain) there is only one value of **y** (in its range). The variable **y** is called **dependent variable** and variable **x** is called **independent variable**. This is written as **$y = f(x)$** .

Example 1:

The order set $\{(1, 2), (3, 2), (-1, -1), (0, 0), (4, 2)\}$ belongs to a function since there is no two or more sets of points with same **x** and different **y**'s.
(to define a function, it is fine to have same values of **y** but not the same values of **x**)

Example 2:

The order set $\{(1, 3), (3, 4), (1, -2), (0, 0)\}$ **does not** belong to a function since there is at least two sets of points, $(1, 3)$ and $(1, -2)$, with same value of **x** and different values of **y**.

Example 3:

$y = f(x) = -2x^3 + 0.5x^2 + 3.5$ is a function. In fact, $f(x)$ is a polynomial of **degree 3** and **leading coefficient - 2**.

Example 4:

Equations of all straight lines are functions except vertical lines. For example, $y = f(x) = 3x - 5$ is a function since it satisfies the definition of a function. In general, straight lines are either written in **standard form**: $ax + bx + c = 0$, where **a**, **b**, and **c** are real numbers, or in the **Slope Intercept form**: $y = f(x) = mx + b$. In the latter form, **m** is called **slope** and **b** is the **y-intercept**. The line is **increasing** if **m > 0**, it is **decreasing** if **m < 0**, and it is **constant** if **m = 0**.

Note:

It is also possible to test for a function if the graph of the equation is given. This is done by vertical line test as follow:

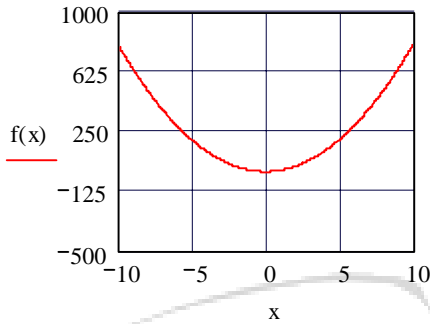
A given curve is the graph of function $y = f(x)$ if and only if any vertical line intersects the graph at only one point.

The following are examples of such vertical line test.

Example 5 :

Graph of a function in XY-Coordinate Systems.
(this is a function)

$$f(x) := 8x^2 + 2x - 1$$

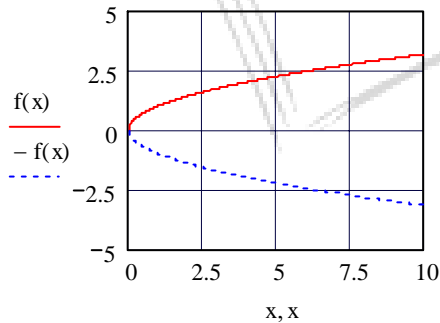


Any vertical line test crosses this graph at only one point, in full agreement with the definition of function.

Example 6:

Graph of a circle here in XY-Coordinate systems.
(this not a function).

$$f(x) := x^{0.5}$$



Both equations, $f(x)$ and $-f(x)$ are functions but combining them as $x = y^2$ result in an equation which is not a function. Any vertical line test crosses this equation in two points, violating the definition of a function.

Remark 1:

Rules of absolute value:

$$|X| = X \quad X \geq 0$$

$$|X| = -X \quad X < 0$$

Example 7:

$$|8| = 8$$

$$|-8| = 8$$

$$|2 - \sqrt{5}| = \sqrt{5} - 2$$

$$|\pi - 4| = 4 - \pi$$

Remark 2:

1. A function $f(x)$ is called **increasing** on a given interval $[a, b]$ if the following conditions exist:

$$f(x_2) > f(x_1) \text{ for any } x_2 > x_1 \text{ belonging to the open interval } (a, b)$$

2. A function $f(x)$ is called **decreasing** on a given interval $[a, b]$ if the following conditions exist

$$f(x_2) > f(x_1) \text{ for any } x_2 < x_1 \text{ belonging to the open interval } (a, b).$$