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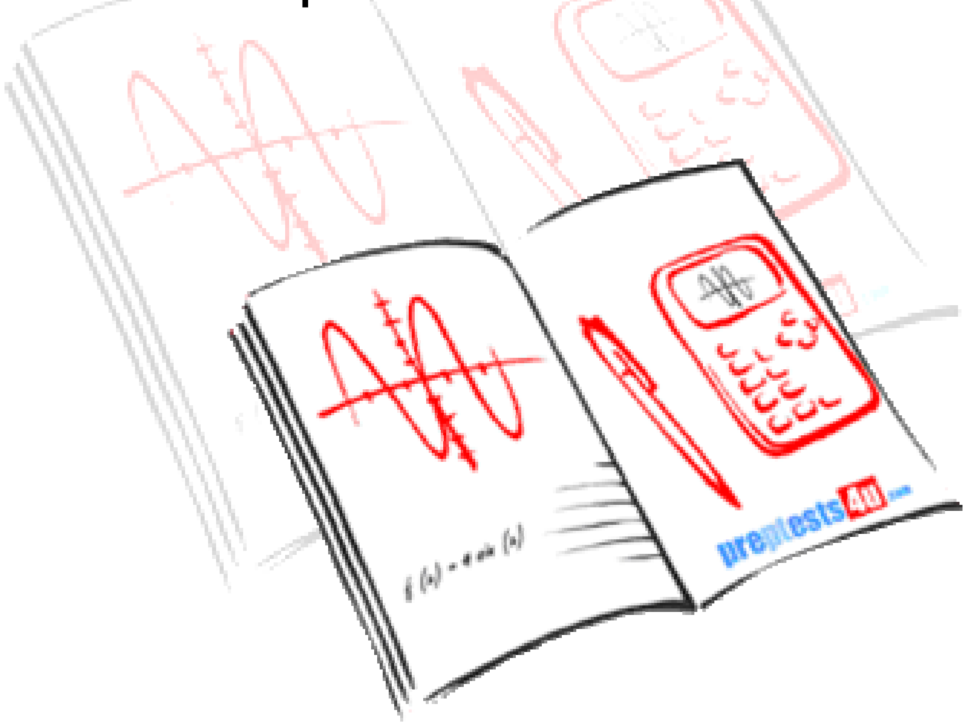
# Business Calculus 1

## Chapter 3 - The Concept of Differentiation

### The Chain Rule and Implicit Differentiation



# The Chain Rule and Implicit Differentiation

Assuming $y = f(g)$ and $g = g(x)$, hence $y = f(g(x))$ and the derivative of $y$ with respect to $x$, called chain rule, is as follow:

$$\frac{dy}{dx} = \frac{dy}{dg} * \frac{dg}{dx}$$

**Example 1:**

Given

$$h(x) = (f \circ g)(x) = f(g(x)) \text{ and } g(4) = 2, g^{'}(4) = 1, f^{'}(2) = 5, f(4) = 6$$

Find $h(4) = ?$

**Solution:**

$$h^{'}(x) = \frac{dh}{dx} = \frac{df}{dg} * \frac{dg}{dx}$$

$$h^{'}(4) = f^{'}(g(4)) * g^{'}(4) = f^{'}(2) * g^{'}(4) = 5 * 1 = 5$$

**Example 2:**

Given $f(x) = 2Cosu + 1$ and $u = \sqrt{1+x}$, find $f^{'}(x)$.

Using Chain Rule formula:

$$\frac{df}{dx} = \frac{df}{du} * \frac{du}{dx} = (-2Sinu) * \frac{1}{2\sqrt{1+x}} = \frac{-2Sinu}{2\sqrt{1+x}} = \frac{-Sin\sqrt{1+x}}{\sqrt{1+x}}$$

**Exercise: Find** $f^{'}(-1)$

## Implicit Differentiation

**Example 3:**

Find the equation of the tangent line to the curve $xy^2 + 2 = \frac{x^2}{y}$ at $(2,1)$.

**Solution:**

Before finding the derivative of the given equation, make sure that the given point is on the curve; hence, substitute $x$ and $y$ values in the equation. Next take the derivative of both sides of the equation with respect to $x$ as follow: Multiplying both sides by $y$ and taking derivative,

$$xy^3 + 2y = x^2$$

$$(1)(y^3) + (x)(3y'y^2) + 2(y') = 2x$$

Rearranging the above equation and solving for $y'$,

$$y' = \frac{2x - y^3}{2 + 3xy^2}$$

$$Slope = m = y'(2,1) = \frac{2 * 2 - 1^3}{2 + 3 * 2 * 1^2} = \frac{3}{8}$$

Equation of the tangent line is:

$$y - 1 = \frac{3}{8}(x - 2) \Rightarrow y = \frac{3}{8}x + \frac{1}{4}$$

**Example 4:**

Find the derivative of the following equation and determine the slope and equation of tangent line at $(0,0)$.

$$y^4 - x^3y^2 = x - x^2y^3$$

**Solution:**

Taking derivative of both sides with respect to $x$,

$$4y'y^3 - 3x^2y^2 - 2y'yx^3 = 1 - 2xy^3 - 3y'y^2x^2$$

Rearranging the terms and solving for $y'$,

$$y' = \frac{dy}{dx} = \frac{1 - 2xy^3 + 3x^2y^2}{4y^3 - 2yx^3 + 3y^2x^2}$$

$$Slope = m = y'(0,0) = \frac{1}{0} = \infty = undefined$$

**Equation of the tangent line at $(0,0)$ with slope $\infty$ is the line $x = 0$, or Y-Axis.**

## Example 5:

Find the derivative of the given equation and show if the function is differentiable at point $(0,0)$

$$Sin(xy) = e^{(xy)}$$

**Solution:**

$$yCosxy + xy'Cosxy = ye^{xy} + xy'e^{xy}$$

$$y' = \frac{dy}{dx} = \frac{ye^{xy} - yCosxy}{xCosxy - xe^{xy}} = \frac{-y}{x}$$

**Exercise: Check for the differentiability of the given function at $(0,0)$.**