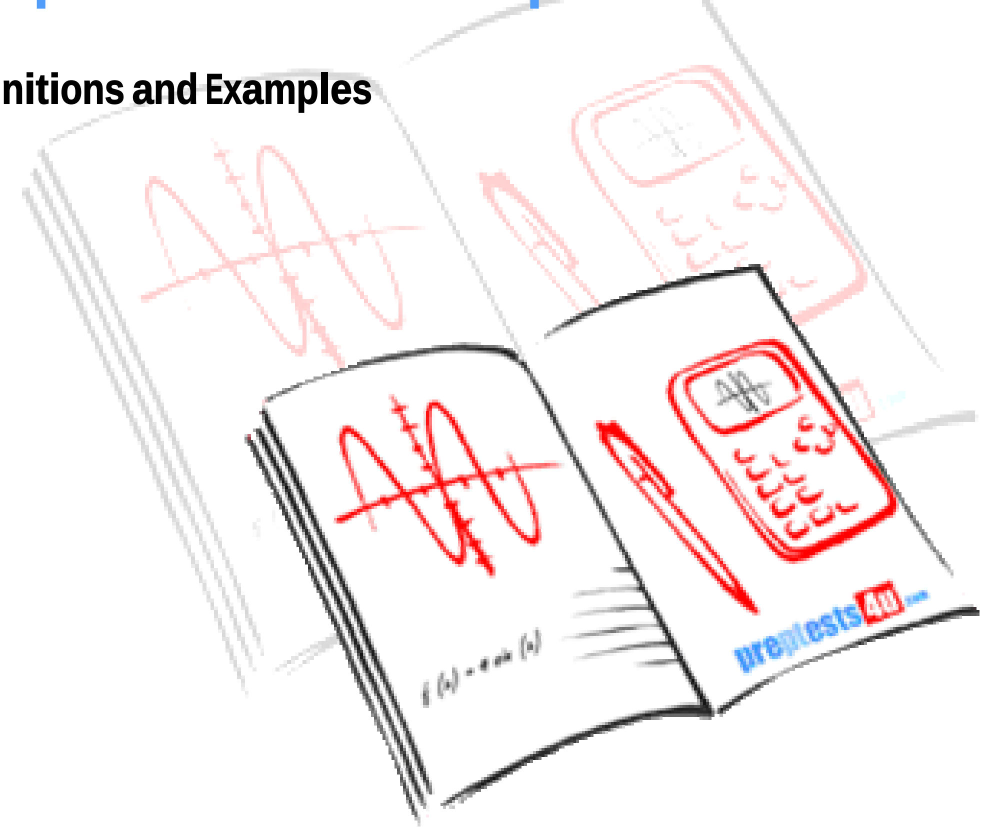
preptests4u.com

Business Calculus 1

Chapter 3 - The Concept of Differentiation

Definitions and Examples



Definition and Examples

The concept of Limit, introduced in Chapter 2, is used to define the derivative of a function as:

The derivative of a function $y = f(x)$ with respect to the independent variable x at a point $x = a$ is:

$$f'(a) = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$$

or in general form,

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

provided such limit exists at a given value(s) of x. Notation h is also used instead of Δx.

Note:

$f'(a)$ **is called slope of the tangent line to the function** $f(x)$ **at point** $(a, f(a))$**.**

Example 1:

Find the derivative of $f(x) = x^2 - x + 1$ at $x = 2$ using definition.

Solution:

$$f(x+\Delta x) = (x+\Delta x)^2 - (x+\Delta x) + 1$$

$$f(x) = x^2 - x + 1$$

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x * \Delta x - \Delta x}{\Delta x} = 2x - 1$$

$$f'(2) = (2*2) - 1 = 3$$

This is the slope of tangent line at point $(2,3)$.
Equation of the tangent line at this point is:

$$y - 3 = 3(x-2)$$

or

$$y = 3x - 3$$

Example 2:

Find equation of a tangent line to function $f(x) = x^2 - x + 1$ at point $(2,1)$.

Solution:
As you notice, $x = 2$ but the point **is not** on the curve $f(x)$, hence the previous rule does not apply.

Note:
If a point with x and y values is given, then you must check if the point belongs to $f(x)$ or not before any attempt to find the slope or /and tangent line.

Example 3:

Find derivatives of $f(x) = \frac{x-1}{2x+1}$ at points $x = 1, -\frac{1}{2}$

Solution:

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = \frac{(x+\Delta x)-1}{2(x+\Delta x)+1}$$

$$f(x) = \frac{x-1}{2x+1}$$

$$f'(x) = \frac{dy}{dx} = \frac{3}{(2x+1)^2}$$

$$f'(1) = \frac{3}{(2+1)^2} = \frac{1}{3}$$

$$f'\left(-\frac{1}{2}\right) = \frac{3}{(-1+1)^2} = \frac{3}{0} = \infty = undefined$$

Note:

The line $x = -\frac{1}{2}$ is called vertical asymptote and the point $x = -\frac{1}{2}$ is called an infinite or non-removable discontinuity. Also, line $y = \frac{1}{2}$ is called horizontal asymptote.

Example 4:

Find derivative of $f(x) = Sinx$ using definition of derivative.

Solution:

$$f(x+\Delta x) = Sin(x+\Delta x)$$

$$f(x) = Sin(x)$$

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{Sin(x + \Delta x) - Sin(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(Sinx)(Cos\Delta x) + (Cosx)(Sin\Delta x) - Sinx}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(Sinx)(1) + (Cosx)(Sin\Delta x) - Sinx}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(Cosx)(Sin\Delta x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(Sin\Delta x)}{\Delta x} * Cosx = Cosx$$

Note:

$$\lim_{\Delta x \to 0} \frac{Sin\Delta x}{\Delta x} = 1$$

The same approach could be used to verify the derivatives of other Trigonometric Functions.