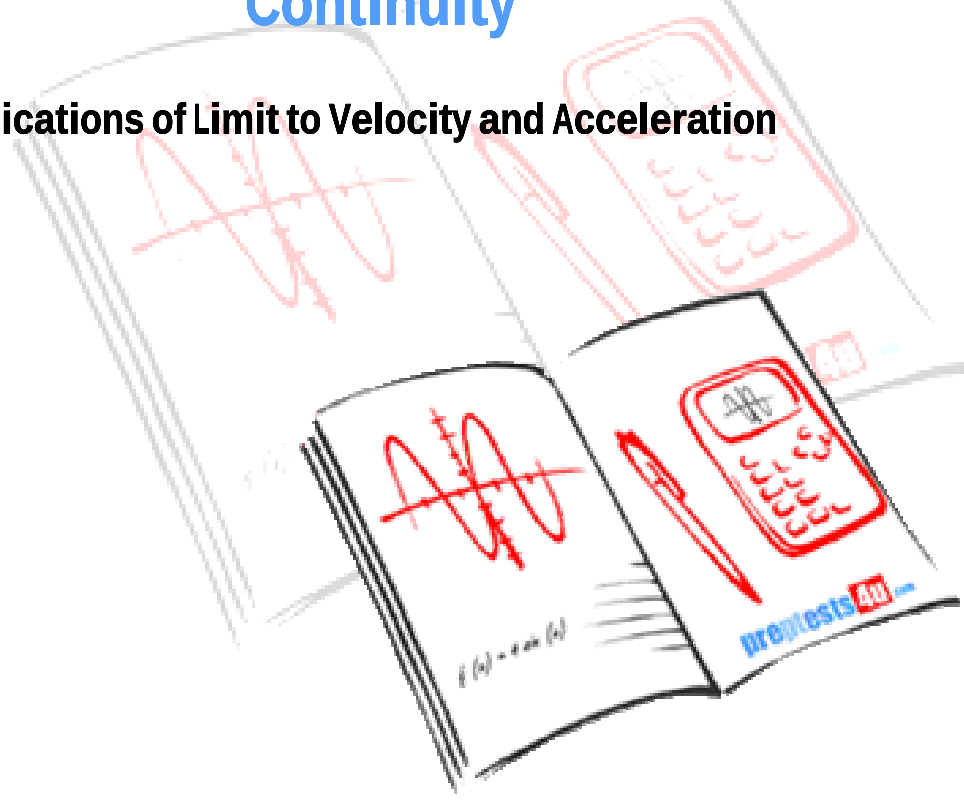
Business Calculus 1

Chapter 2 - The Concepts of Limit and Continuity

Applications of Limit to Velocity and Acceleration



Applications of Limit to Velocity and Acceleration

In this section, we concentrate on the Rectilinear Motion, motion on a straight line, and postponed the complete account of motion, velocity, and acceleration, using derivatives, to the next chapter.

A particle is moving on a straight line having the distance traveled $s = f(t), t \geq 0$.

a. **Average Velocity**

$$v_{ave} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}, t_2 \geq t_1$$

b. **Instantaneous Velocity**

$$v(t) = \lim_{(t_2 - t_1) \to 0} \frac{f(t_2) - f(t_1)}{t_2 - t_1}, t_2 \geq t_1$$

In this case, $v(t) = \frac{ds}{dt} = \frac{df(t)}{dt}$

c. **Average Acceleration**

$$a_{ave} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}, t_2 \geq t_1$$

d. **Instantaneous Acceleration**

$$a(t) = \lim_{(t_2 - t_1)\to 0} \frac{v(t_2) - v(t_1)}{t_2 - t_1}, t_2 \geq t_1$$

In this case, $v(t) = \frac{ds}{dt} = \frac{df(t)}{dt}$ and $a(t) = \frac{dv}{dt} = \frac{d^2 f(t)}{dt^2}$

Note:
To find the velocity or / and acceleration, we use formulas **b.** and **d.**, respectively, unless otherwise stated. The applications of these formulas are given in the following example.

Example 1:

A ball is dropped from a tower of height $h_\circ$ above the ground. Find:

a. Its average velocity between $t = 1s$ and $t = 3s$

Solution:

This is a free fall case where the ball is falling under the force of gravity. General formula for a free fall is given;

$$h(t) = h_\circ + v_\circ t + \frac{1}{2} g t^2$$

where $v_\circ$ is the initial velocity, $g = 9.8 m/s^2 = 32 ft/s^2$ is the acceleration due to force of gravity, and t is the travel time. The ball is falling vertically, on a straight line; hence formulas of rectilinear motion are applicable.

Since the ball had no initial velocity, stationary on the top of the tower, $v_\circ = 0$.
Taking the point of reference to be where the ball was before dropping, $h_\circ$ will take a negative value since the ground is below the reference point, negative y-axis. Therefore,

$$h(3) = 0 + 0 + \frac{1}{2}\left(9.8m/s^2\right)\left(3^2\right)$$

$$h(1) = 0 + 0 + \frac{1}{2}\left(9.8m/s^2\right)\left(1^2\right)$$

$$v_{ave} = \frac{h(3) - h(1)}{3 - 1} = 19.6m/s$$

b. The distance traveled after 4 seconds.

Solution:

$$h(4) = 0 + 0 + \frac{1}{2}\left(9.8m/s^2\right)\left(4s\right)^2 = 78.4m$$

Note:
Assume that $h_\circ \succ 78.4m$

c. The speed of the ball when it hits the ground.

Solution:

When the ball hits the ground, $h = 0$. Therefore,

$h_\circ = \frac{1}{2}gt^2$ and $v = gt$.

Since $t = \sqrt{\frac{2h_\circ}{g}}$,

$$v_{ground} = g * \sqrt{\frac{2h_\circ}{g}} = \sqrt{2g * h_\circ} = \sqrt{9.8h_\circ}\, m/s$$

d. Find average acceleration between $t = 1s$ and $t = 3s$.

Solution:

$v(1) = g \quad m/s^2$ and $v(3) = 3g \quad m/s^2$

$$a_{ave} = \frac{v(3) - v(1)}{3 - 1} = g = 9.8m/s^2$$

Note:
As you see from this calculation, the average acceleration equals to g.