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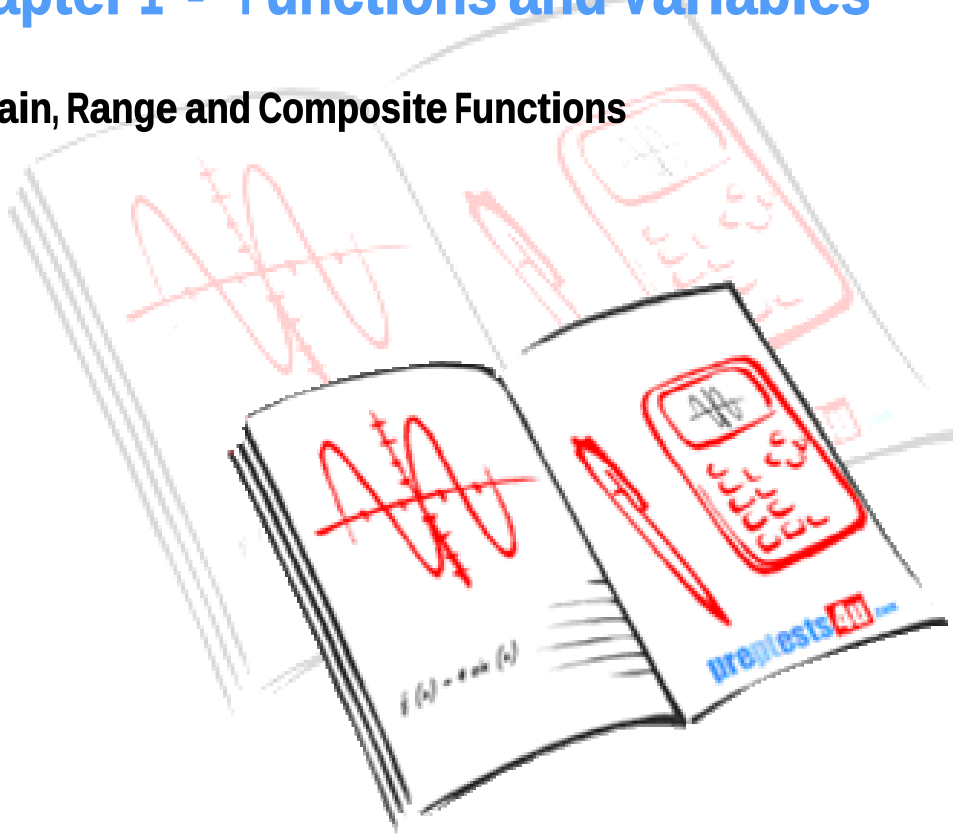
# Business Calculus 1

## Chapter 1 - Functions and Variables

### Domain, Range and Composite Functions



# Domain, Range and Composite Functions

Suppose $y = f(x)$ is a function of single variable where 'x' is an independent variable and 'y' is a dependent variable.

**Domain** of this function is defined as all real values of 'x' in which $f(x)$ is defined or all those values of 'x' which result in finite values of 'y'.

Since Range of the function is dependent on the Domain, one must determine the domain first. However, finding the Range is not as trivial as finding the Domain.
**Range** of this function is the set of all values of 'y', where these values are generated by the values of 'x' in its domain.

The following examples will illustrate the method of finding **Domain** and **Range** for a given function.

1. Domain of all straight lines, excluding vertical lines, is ( -∞, +∞)
2. Range of all straight lines is, except horizontal lines, ( -∞, +∞)
3. Domain of all polynomials is ( -∞, +∞)
4. Range of all polynomials is ( -∞, +∞)

## Example 1:

Find the domain and range for the function: $f(x) = \sqrt{x^2 - 16}$

1. Domain: In order to have f (x) defined, we must have $x^2 - 16 \geq 0$

$$(x-4)(x+4) \geq 0$$
$$(x-4) \text{ or } (x+4) \geq 0$$
$$x \geq 4 \text{ or } x \leq -4$$

**Domain:** $(-\infty, -4] U [+4, +\infty)$

2. Range: The range for this function is $[0, +\infty)$ since minimum value of 'y' is zero and it increases as the magnitude of 'x' exceed four.

**Range:** $[0, +\infty)$

**Example 2:**

Find the domain and range for the function: $f(x) = \dfrac{x^2 + 4x - 6}{\sqrt{x^2 - 16}}$

Domain for the numerator is ( - ∞, ∞ ). This is a polynomial of degree 2.
Domain for the denominator is the same as Example 1: $(-\infty, -4]U[+4, +\infty)$.

Since the values x = -- 4 and x = + 4 make the denominator zero resulting in $f(x)$ undefined, they must be excluded from the $f(x)$ domain.

So the Domain of $f(x)$ is:

**Domain** $(-\infty, -4) \mathrm{U} (4, \infty)$

**Exercise: Find the range for this function.**

**Example 3:**

Find the domain and range for the function: $f(x) = \dfrac{\sqrt{1-x}}{x^2 - 4}$

In order to make the function defined, the following conditions must be satisfied:

$1 - x \geq 0$ and $x^2 - 4 \neq 0$

$(x-2)(x+2) \neq 0$

Therefore, $x \leq 1$ and $x \neq 2, -2$

So, the domain of $f(x)$ is:

**Domain:** $(-\infty, -2) \cup (-2, 1)$

**Note:**
**To find the range, one may graph the function $f(x)$ and find the pattern in which the values of 'y' are changing.**

## Composite Functions

Given two functions $f(x)$ and $g(x)$, the composite functions of $f \circ g$ and $g \circ f$ are defined as follow:

$$(f \circ g)(x) = f(g(x)) \text{ and } (g \circ f)(x) = g(f(x))$$

If $f \circ g = g \circ f = x$, then functions $f(x)$ and $g(x)$ are inverse of each other.

**Note:**
**In general** $(f \circ g)(x) \neq (g \circ x)(x)$

### Example 4:

Given: $f(x) = 2x - 1 \qquad g(x) = 1 - x^2$

Find:

a. $f \circ g = f(g(x)) = 2(g(x)) - 1 = 2(1 - x^2) - 1 = 1 - 2x^2$

b. $g \circ f = g(f(x)) = 1 - (g(x))^2 = 1 - (2x - 1)^2 = -4x^2 + 4x$

**Note:**
$(f \circ g)(x) \neq (g \circ x)(x)$

### Example 5:

Given: $f(x) = \dfrac{1}{2 - 3x}$ and $g(x) = \dfrac{\sqrt{x - 1}}{x^2 + 1}$

Find:

a. $f \circ g = f(g(x)) = \dfrac{1}{2 - 3g(x)} = \dfrac{1}{2 - 3\dfrac{\sqrt{x-1}}{x^2+1}} = \dfrac{x^2 + 1}{2(x^2 + 1) - 3\sqrt{x - 1}}$

**Note:**
**Replace all x's in** $f(x)$ **with** $g(x)$ **and simplify**

b. $g \circ f = g(f(x)) = \dfrac{\sqrt{f(x)-1}}{(f(x))^2+1} = \dfrac{\sqrt{\dfrac{1}{2-3x}-1}}{\left(\dfrac{1}{2-3x}\right)^2-1}$

**Remark:**

**There is no reason to do further work unless a value of $x \in$ domain of $f(x)$ is given.**

c. $f \circ f = f(f(x)) = \dfrac{1}{2-3f(x)} = \dfrac{1}{2-3\left(\dfrac{1}{2-3x}\right)} = \dfrac{2-3x}{1-6x}$

d. Try to find $g \circ g$ if you are not very busy or believe that these questions have no use in real life.

**Example 6:**

Given: $f(x) = \dfrac{5}{x-2}$ and $g(x) = |-x|$

Find:

a. $(f \circ g)(x) = f(g(x)) = \dfrac{5}{g(x)-2} = \dfrac{5}{|-x|-2} = \dfrac{5}{|x|-2}$

$|x| = x, x \geq 0$ and $|x| = -x, x < 0$.

The domain for $f \circ g$ is $(-\infty,-2)U(2,+\infty)$.

b. $(g \circ f)(x) = g(f(x)) = |-f(x)| = \left|-\dfrac{5}{x-2}\right| = \left|\dfrac{5}{x-2}\right| = \dfrac{|5|}{|x-2|} = \dfrac{5}{|x-2|}$

The domain for $g \circ f$ is all $x \neq 2$ or $(-\infty,2)U(2,+\infty)$.

**Note:**

**Domain of $f \circ g$ is not the same as that of $g \circ f$ .**

c. $g \circ g = g(g(x)) = \left\| -x \right\| = \left\| x \right\| = |x|$

Domain for $g \circ g$ is all real values of $x$ or $(-\infty, +\infty)$.
Range for $g \circ g$ is $[0, +\infty)$

d. $f \circ f = f(f(x)) = \dfrac{5}{f(x) - 2} = \dfrac{5}{\dfrac{5}{x-2} - 2} = \dfrac{5(x-2)}{-2x+9} = \dfrac{5x-10}{-2x+9}$

Domain for $f \circ f$ is all real values of $x$ but $\frac{9}{2}$ or $\left(-\infty, \frac{9}{2}\right) U \left(\frac{9}{2}, +\infty\right)$.

Try to find the Range for $f \circ f$ .